Quiz 1 Solution

Tuesday, January 24, 2023 12:52 PM

1. (*25 points*) For the planar system consisting of two rigid links AB and BC connected using smooth pins to each other and to the supports, find the angular velocity of the link BC and the linear velocity $\mathbf{V}_C$ of the sliding support C if $\omega_{AB} = 2rad/s$ (counter clockwise). Assume that the support C can smoothly translate in the horizontal directions, but is constrained against motion in the vertical direction.

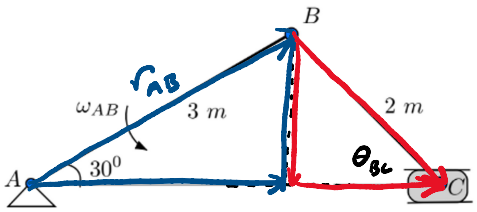


Figure 1: Problem 1

Vector Method

$$\bar{V}_C = V_B + \omega_{BC} \times r_{BC}$$

$\longrightarrow$

$$V_B = \omega_{AB} \times r_{AB}$$

$$\omega_{AB} = 0\,\hat{i} + 0\,\hat{j} + 2\,\hat{k}$$

$$r_{AB} = 3\cos(30)\,\hat{i} + 3\sin(30)\,\hat{j}$$

$$r_{AB} = \frac{3\sqrt{3}}{2}\,\hat{i} + 1.5\,\hat{j}$$

$$V_B = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 2 \\ \frac{3\sqrt{3}}{2} & 1.5 & 0 \end{vmatrix} = -3\,\hat{i} + 3\sqrt{3}\,\hat{j}$$

$$\omega_{BC} = 0\,\hat{i} + 0\,\hat{j} - \omega_{BC}\,\hat{k}$$

$$r_{BC} = 2\cos(\theta_{BC})\,\hat{i} - 2\sin(\theta_{BC})\,\hat{j}$$

$\theta_{BC} = ?$

$$\sin(30) = \frac{h}{3}$$

$$\sin(\theta_{BC}) = \frac{h}{2}$$

$$\sin\theta_{BC} = \frac{3\sin(30)}{2}$$

$$\sin\theta_{BC} = \frac{1.5}{2} = 0.75$$

$$\theta_{BC} = 48.59°$$

$$r_{BC} = 2(0.6614)\hat{i} - 1.5\hat{j}$$

$$r_{BC} = 1.3229\hat{i} - 1.5\hat{j}$$

$$\omega_{BC} \times r_{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -\omega_{BC} \\ 1.32 & -1.5 & 0 \end{vmatrix} = -1.5\omega_{BC}\hat{i} - 1.32\omega_{BC}\hat{j}$$

$$v_C = -3\hat{i} + 3\sqrt{3}\hat{j} - 1.5\omega_{BC}\hat{i} - 1.32\omega_{BC}\hat{j}$$

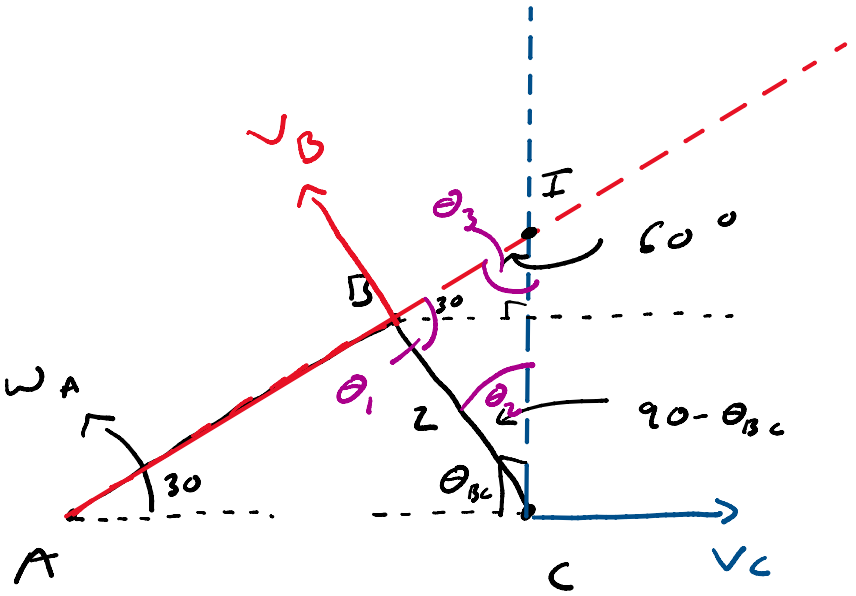
no movement in y

$$3\sqrt{3} - 1.32\omega_{BC} = 0$$

$$\omega_{BC} = 3.93 \text{ rad/s}$$ ✓

$$v_C = -8.89\hat{i} \text{ m/s}$$ ✓

Instantaneous Center



$\theta_2 = 90 - 48.59 = 41.41°$

$\theta_3 = 60°$ $\quad \theta_1 = 180 - 60 - 41.41 = 78.59°$

Law of sines to find r_{IB} & r_{IC}

$$\frac{2}{\sin\theta_3} = \frac{r_{IB}}{\sin(\theta_2)} \qquad r_{IB} = \frac{2\sin(\theta_2)}{\sin(\theta_3)} = 1.5275$$

$$\frac{2}{\sin\theta_3} = \frac{r_{IC}}{\sin\theta_1} = \qquad r_{IC} = \frac{2\sin(\theta_1)}{\sin(\theta_3)} = 2.2638$$

$$v_B = r_{IB}\,\omega_{BC} = r_{AB}\,\omega_{AB}$$

$$\omega_{BC} = \frac{r_{AB}\,\omega_{AB}}{r_{IB}} = \frac{3(2)}{1.5275} = \boxed{3.9280 \text{ rad/s}}$$ ✓

$$v_C = r_{IC}\,\omega_{BC} = (2.2638)(3.9280) = \boxed{8.8922 \text{ m/s}}$$ ✓